AMERICAN UNIVERSITY OF BEIRUT
Department of Electrical and Computer Engineering
EECE340 Signals and Systems
Section 2

Lecturer: Prof. Fadi N Karameh

Quiz 2- May 17, 2012

Directions:

- You have 2.0 hours for this exam.
- Write down your initials *in ink* on all the pages.
- Answers must be explained or derived. DO NOT just write down an answer, unless otherwise indicated.
- It is a good idea to read the whole test before you begin. Problems are divided into several parts with percentages indicated. You might be able to solve different parts independently.
- DO NOT talk to any of your colleagues under any circumstances. You will be penalized without warning.

YOUR NAME HERE: SOLUTIONS, MR.

Problem 1 ________

Problem 2 ________

Problem 3 ________

Problem 4 ________

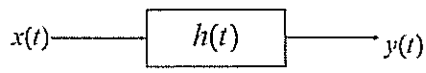
TOTAL

PROBLEM 1 (22%) (DIFFERENT PARTS OF THIS PROBLEM ARE INDEPENDENT)

a) (8%) For the CT LTI system shown in figure 1 below, find the output $y(t)$ if an input

$$x(t) = e^{-3t}u(t) + e^{-t}u(t)$$

is applied.



$$h(t) = \delta(t) - e^{-|t|}$$

Figure 1:

$y(t) = h(t) * x(t)$

or $Y(s) = H(s)\,X(s)$ $\qquad X(s) = \dfrac{1}{s+3} + \dfrac{1}{s+1}$

$h(t) = \delta(t) - e^{-t}u(t) - e^{t}u(-t) \Rightarrow H(s) = 1 - \dfrac{1}{s+1} + \dfrac{1}{s-1} = 1 + \dfrac{2}{(s+1)(s-1)}$

$\therefore Y(s) = \dfrac{1}{s+1} + \dfrac{1}{s+3} + \dfrac{2}{(s+1)(s-1)(s+3)} + \dfrac{2}{(s+1)^2(s-1)}$

Use Partial Fraction expansion

$$\frac{2}{(s+1)(s-1)(s+3)} = \frac{-\frac{1}{2}}{s+1} + \frac{\frac{1}{4}}{s-1} + \frac{\frac{1}{4}}{s+3}$$

$$\frac{2}{(s+1)^2(s-1)} = \frac{-1}{(s+1)^2} - \frac{\frac{1}{2}}{s+1} + \frac{\frac{1}{2}}{s-1}$$

$\checkmark\ Re(s) > -3 \qquad Re(s) < 1 \qquad \checkmark\ Re(s) > -1$

$$\therefore Y(s) = \frac{\frac{5}{4}}{s+3} + \frac{\frac{3}{4}}{s-1} - \frac{1}{(s+1)^2}$$

$$\Rightarrow \left[\, y(t) = \frac{5}{4}e^{-3t}u(t) - \frac{3}{4}e^{t}u(-t) - te^{-t}u(t) \,\right]$$

b) (8%) For the DT LTI system shown in figure 2 below, find the output $y[n]$ if an input

$$x[n] = \left(\frac{1}{2}\right)^{n} u[n] + \left(\frac{2}{3}\right)^{-n} u[-n-1]$$

is applied.

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$h[n] = \left(\frac{3}{4}\right)^{|n|}$$

Figure 2:

$$h[n] = \left(\frac{3}{4}\right)^{n} u(n) + \left(\frac{4}{3}\right)^{n} u(-n-1)$$

$$Y(z) = H(z) \cdot X(z)$$

$$H(z) = \frac{1}{1-\frac{3}{4}z^{-1}} - \frac{1}{1-\frac{4}{3}z^{-1}}$$

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-\frac{3}{2}z^{-1}}$$

$$Y(z) = H(z) \cdot X(z)$$

$$H(z) = \frac{1-\frac{4}{3}z^{-1} - 1 + \frac{3}{4}z^{-1}}{\left(1-\frac{3}{4}z^{-1}\right)\left(1-\frac{4}{3}z^{-1}\right)} = \frac{-\frac{7}{12}z^{-1}}{\left(1-\frac{3}{4}z^{-1}\right)\left(1-\frac{4}{3}z^{-1}\right)}$$

$$X(z) = \frac{1-\frac{3}{2}z^{-1} - 1 + \frac{1}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{3}{2}z^{-1}\right)} = \frac{-z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{3}{2}z^{-1}\right)}$$

$$Y(z) = \frac{\frac{7}{12}z^{-2}}{\left(1-\frac{3}{4}z^{-1}\right)\left(1-\frac{4}{3}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{3}{2}z^{-1}\right)}$$

ignore z^{-2} then delay by 2 steps.

Using PFE:

$$Y(z) = \frac{\frac{27}{12}}{1-\frac{3}{4}z^{-1}} - \frac{\frac{256}{15}}{1-\frac{4}{3}z^{-1}} + \frac{\frac{7}{20}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{63}{4}}{1-\frac{3}{2}z^{-1}}$$

$$\therefore y(n) = \frac{9}{4}\left(\frac{3}{4}\right)^{n-2} u(n-2) + \frac{256}{15}\left(\frac{4}{3}\right)^{n-2} u[-n+1] - \frac{7}{20}\left(\frac{1}{2}\right)^{n-2} u(n-2) - \frac{63}{4}\left(\frac{3}{2}\right)^{n-2} u(-n+1)$$

c) (6%) Find the energy of the output $y(t)$ for the product operation shown in figure 3 below.

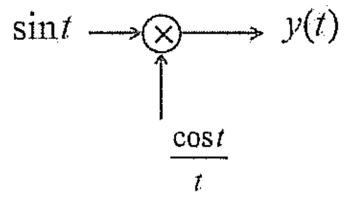
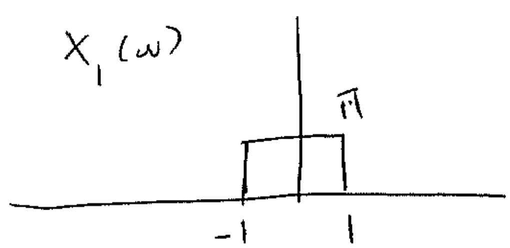


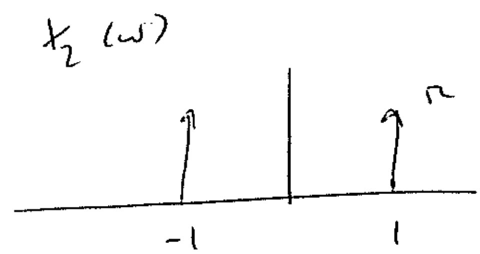
Figure 3:

$$y(\tau) = \sin\tau \cdot \frac{\cos\tau}{\tau} = \frac{\sin\tau}{\tau} \cdot \cos\tau$$

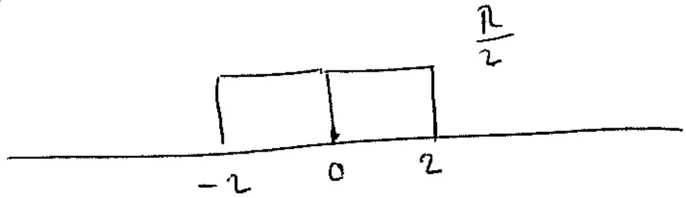
$$E = \int_{-\infty}^{\infty} y^2(\tau)\, d\tau = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y^2(\omega)\, d\omega$$ PARSEVALS.

$$Y(\omega) = \frac{1}{2\pi} X_1(\omega) * X_2(\omega)$$ where $x_1(\tau) = \frac{\sin\tau}{\tau}$, $x_2(\tau) = \cos\tau$





$\therefore Y(\omega)$



$$E = \frac{1}{2\pi}\left(\frac{\pi^2}{4}\right) \cdot 4 = \frac{\pi}{2}$$

EXTRA SPACE FOR PROBLEM 1

PROBLEM 2 (28%) (DIFFERENT PARTS OF THIS PROBLEM ARE INDEPENDENT)

a) (8%) Let $y(t) = h(t) * x(t)$ where $x(t)$ and $h(t)$ are as shown in figure 4 below. Find $y(t)$ at $t = \frac{11}{2}$ sec.

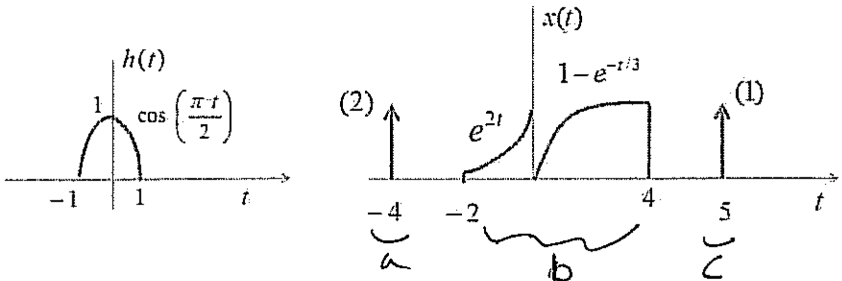
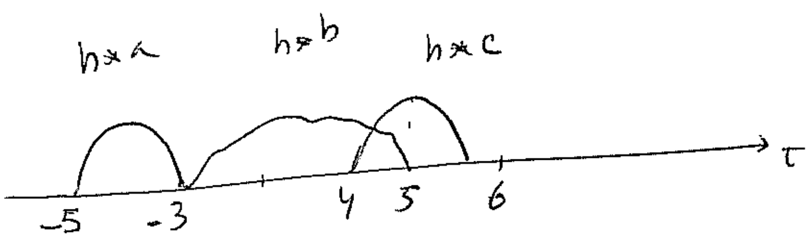


Figure 4:

$y(t) = h(t) * x(t)$ will have the following time support:



therefore for $t = \frac{11}{2} = 5.5$ sec, only $h*c$ will affect the output

$h(t) * \delta(t-5) = h(t-5)$

for $t = 5.5 \rightarrow y(5.5) = h(0.5) = \cos\frac{\pi}{4}$

$= \frac{\sqrt{2}}{2}$

b) (12%) Consider the block diagram representation of CT LTI causal system as shown in figure 5 below.

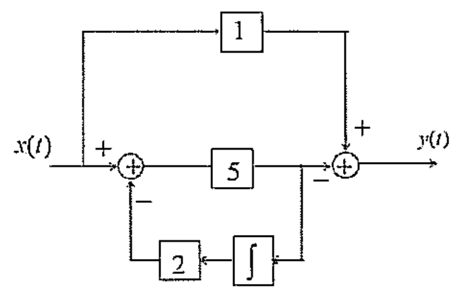


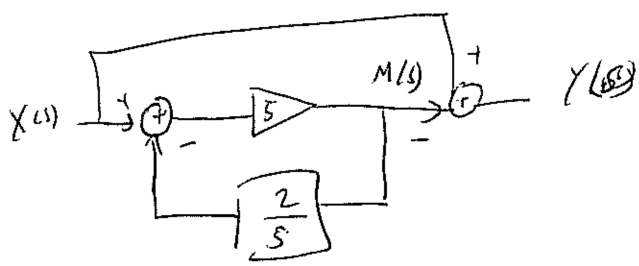
Figure 5:

i- Find the impulse response of this system $h(t)$.

ii- Assume I want to recover the input $x(t)$ from the output $y(t)$ of this system by passing $y(t)$ through another system whose impulse response is given by $h_1(t)$. Determine, if possible, $h_1(t)$.

iii- Is $h_1(t)$ implementable? explain.

i) redraw in S-domain



$$M(s) = \frac{5}{1+\frac{2}{s}} \cdot X(s)$$

$$Y(s) = \left[1 - \frac{5}{1+\frac{10}{s}}\right] X(s) \Rightarrow \frac{Y(s)}{X(s)} = H(s) = \frac{(s+10)-5s}{s+10} = \frac{10-4s}{s+10}$$

$$\therefore h(t) = 10e^{-10t}u(t) + 40e^{-10t}u(t) - 4\delta(t)$$

$$\left[h(t) = 50e^{-10t}u(t) - 4\delta(t)\right]$$

ii $Y(s)$ —— $\boxed{H_1(s)}$ —— $N(s)$

$$\because N(s) = H_1(s) \cdot Y(s) = H_1(s) \cdot H(s) \cdot X(s) = X(s)$$

$$\Rightarrow H_1(s) = \frac{1}{H(s)} = \frac{s+10}{-4s+10} = \frac{-\frac{1}{4}(s+10)}{s-\frac{10}{4}} \Rightarrow h_1(t) = -\frac{1}{4}\delta(t) + 50e^{\frac{10}{4}t}u(-t)$$

$$= \cancel{\tfrac{1}{4} + \frac{5s}{-4s+10}}$$

which is NOT CAUSAL (to be stable) $\Rightarrow$ NOT implementable

EXTRA SPACE FOR PROBLEM 2-b

c) (8%) Assume that a periodic signal $x(t)$ is applied to the system shown in figure 6 below, please determine the corresponding output signal $y(t)$.

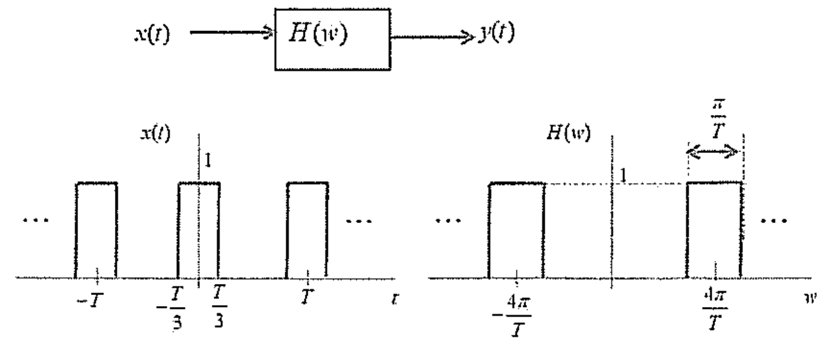
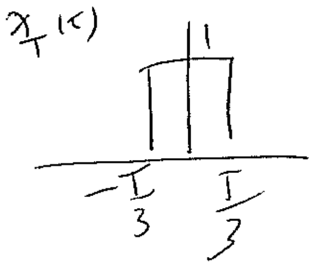


Figure 6:

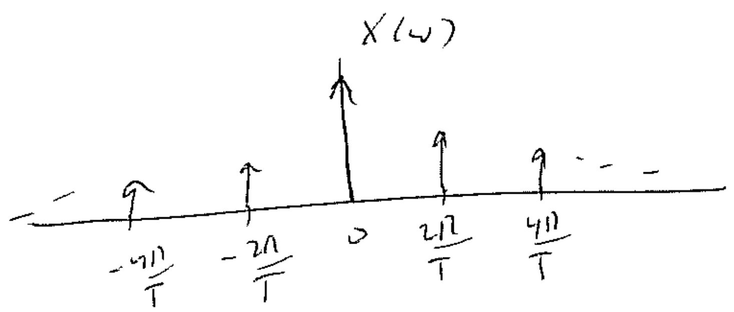


$$\Rightarrow X_T(\omega) = \frac{2T}{3}\,\mathrm{sinc}\left(\frac{\omega T}{3}\right)$$

$$X(\omega) = \frac{2\pi}{T}\sum_K \frac{2T}{3}\,\mathrm{sinc}\left(\frac{2\pi K}{3}\right)\delta\left(\omega - \frac{2\pi}{T}K\right)$$

$$= \frac{4\pi}{3}\sum_K \frac{\sin\frac{2\pi K}{3}}{\frac{2\pi K}{3}}\,\delta\left(\omega - \frac{2\pi}{T}K\right)$$

$$X(\omega) = \frac{4\pi}{3}\sum_K \mathrm{sinc}\frac{2\pi K}{3}\,\delta\left(\omega - \frac{2\pi}{T}K\right)$$



passing through H(ω) only K=2 survives

$$Y(\omega) = \frac{4\pi}{3}\left[\mathrm{sinc}\frac{4\pi}{3}\,\delta\left(\omega - \frac{4\pi}{T}\right) + \mathrm{sinc}\frac{4\pi}{3}\,\delta\left(\omega + \frac{4\pi}{T}\right)\right]$$

$$\Rightarrow y(t) = \frac{4}{3}\,\mathrm{sinc}\left(\frac{4\pi}{3}\right)\cos\frac{4\pi}{T}t$$

$$= \frac{4}{3}\cdot\frac{3}{4\pi}\sin\frac{4\pi}{3}\cos\frac{4\pi}{T}t = -\frac{\sqrt{3}}{2\pi}\cos\frac{4\pi}{T}t$$

EXTRA SPACE FOR PROBLEM 2

PROBLEM 3 (25%)

Consider the AM modulation scheme shown in figure 7 below. The input signal is assumed bandlimited to B rad/sec. Assume that the oscillator frequency $w_o = 10B$ rad/sec

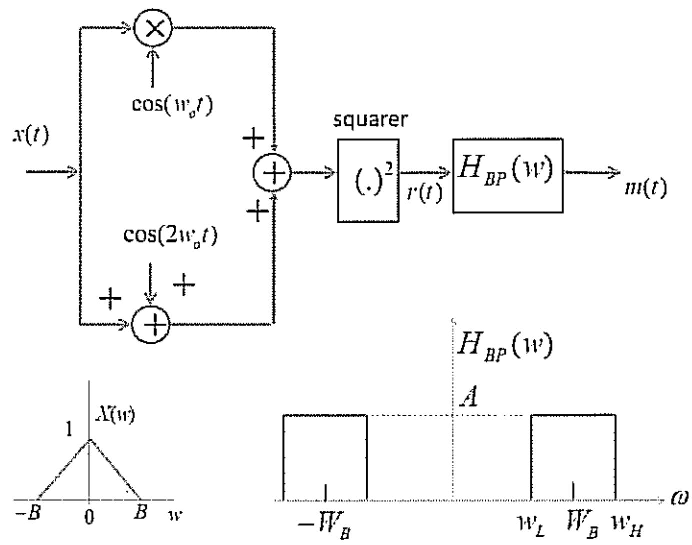


Figure 7:

a) (8%)

i- First, find the signal $r(t)$ in the time domain in terms of the input $x(t)$. NOTE: Be careful with summers and multipliers in the figure!

ii- Second, sketch the corresponding CTFT $R(w)$.

$$r(t) = \left(x \cos w_o t + (x + \cos 2w_o t)\right]^2$$

$$= x^2 \cos^2 w_o t + 2x \cos w_o t\,(x + \cos 2w_o t) + (x + \cos 2w_o t)^2$$

$$= x^2 \cos^2 w_o t + 2x^2 \cos w_o t + 2x \cos w_o t \cos 2w_o t$$
$$+ x^2 + 2x \cos 2 w_o t + \cos^2 2w_o t$$

$$= \frac{x^2}{2} + \frac{x^2 \cos 2w_o t}{2} + 2x^2 \cos w_o t + x \cos w_o t + x \cos 3w_o t$$
$$+ x^2 + 2x \cos 2w_o t + \frac{1}{2} + \cos 4w_o t$$

EXTRA SPACE FOR PROBLEM 3-a

$$r(t) = x^2\left[1 + 2\cos\omega_0 t\right] + x^2\left(1 + \frac{\cos 2\omega_0 t}{2}\right)$$

$$+ x\cos\omega_0 t + 2x\cos\omega_0 t + \frac{1 + \cos 4\omega_0 t}{2} + x(t)\cos 3\omega_0 t$$

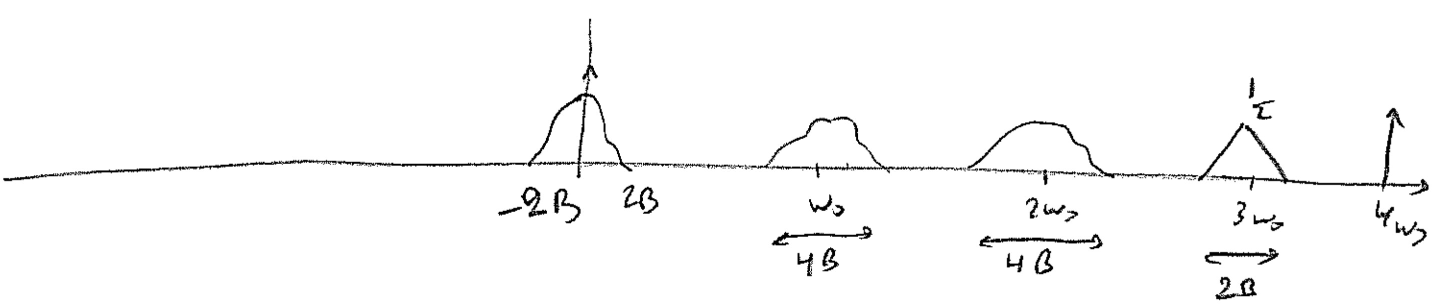
$$= \left(x^2 + \frac{1}{2} + \frac{x^2}{2}\right) + \left(2x^2 + 3x\right)\cos\omega_0 t$$

$$+$$

$$= \frac{3x^2}{2} + \frac{1}{2} + \left(2x^2 + x\right)\cos\omega_0 t + \left(\frac{x^2}{2} + 2x\right)\cos 2\omega_0 t$$

$$+ x\cos 3\omega_0 t + \cos 4\omega_0 t$$

$R(\omega)$



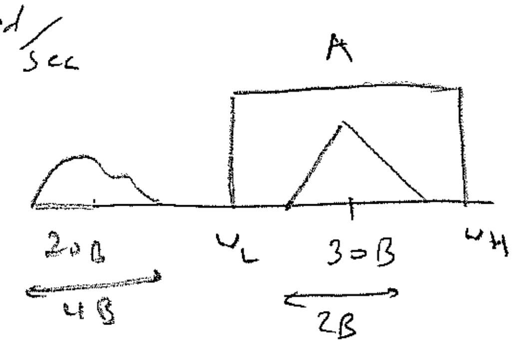
This is the only useful one

We now require that the output signal $m(t)$ be a **DSB-SC AM signal.** To do so, please Answer the following questions.

b) i- (7%) Determine the gain A , the center frequency W_B, and the range for w_L and w_H of the band pass filter for proper operation.

ii- (4%) Sketch the CTFT $M(w)$ in the frequency domain.

$W_B = 3\omega_0 = 30B$ rad/sec



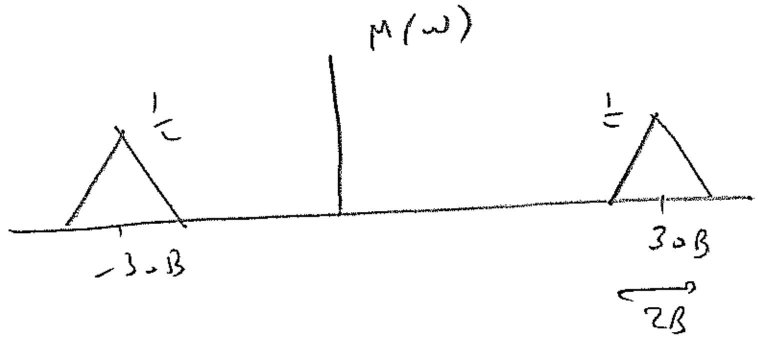
$w_L < 29B$

& $22B = 2w_0 + 2B < w_L$

$\Rightarrow [22B < w_L < 29B]$

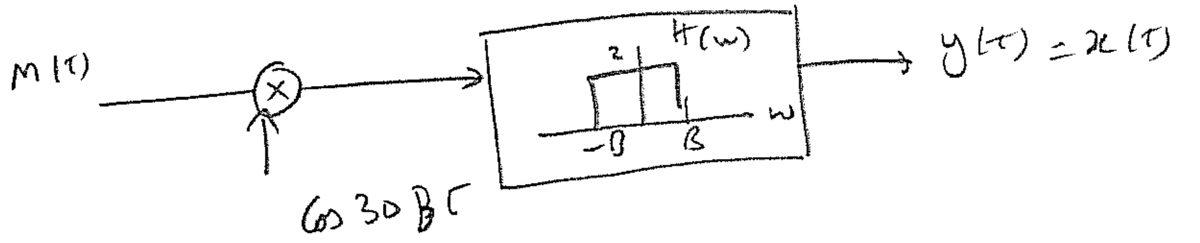
$[w_H > 31B$ & $w_H < 40B]$

gain $A = 1$ (To get the regular DSBSC AM)



EXTRA SPACE FOR PROBLEM 3-b

c) (6%) Suggest a scheme whereby the signal $x(t)$ can be recovered from the DSB-SC AM signal $m(t)$.



PROBLEM 4 (25 %)

Consider the problem of sampling a CT signal which is bandlimited to B rad/sec using the sampler shown in figure 8

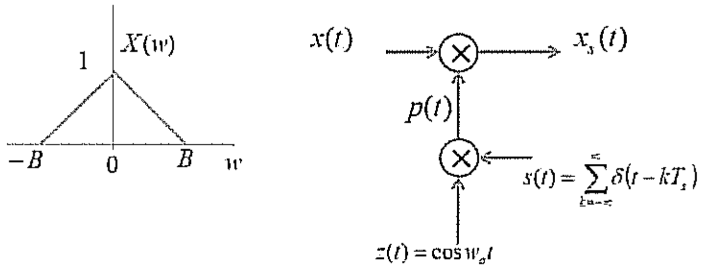


Figure 8:

The sampling signal $p(t)$ is formed by sampling a signal $z(t) = \cos w_o t$ with an ideal picket fence. That is,

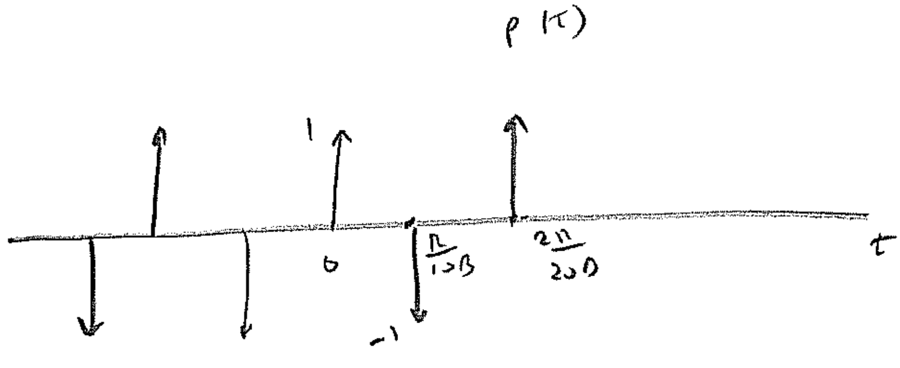
$$p(t) = z(t)s(t) = (\cos w_o t) \sum_{k=-\infty}^{\infty} \delta(t - nT_s)$$

Assume the following values: $w_o = 10B$ rad/sec and $T_s = \frac{2\pi}{20B}$

a) (6 %) Sketch $p(t)$ in the time domain.

$$p(t) = \cos 10Bt \sum \delta\left(t - \frac{2\pi k}{20B}\right)$$

$$= \sum_{k=-\infty}^{\infty} \cos \pi k \, \delta\left(t - \frac{\pi}{10B} k\right)$$

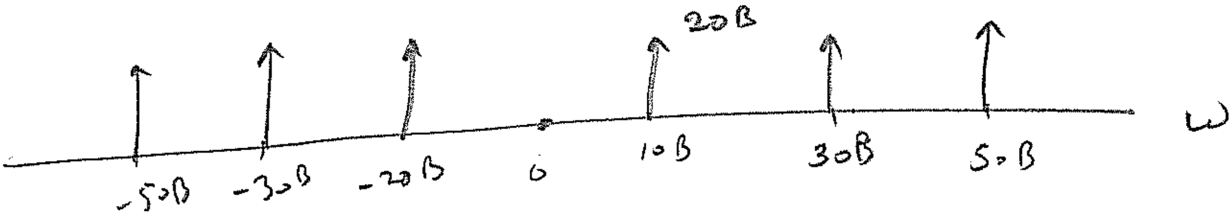


b) (6%) Sketch $P(w)$ the CTFT of the signal $p(t)$. Label the axis carefully as you will use this in subsequent parts.

$$P(\omega) = \frac{1}{2\pi}\left(\pi\,\delta(\omega-10B) + \pi\,\delta(\omega+10B)\right) * 20B \sum_{k=-\infty}^{\infty} \delta(\omega-20Bk)$$

$$= 10B \sum_{k=-\infty}^{\infty} \delta(\omega-10B-20Bk) + \delta(\omega+10B-20Bk)$$

$P(\omega)$



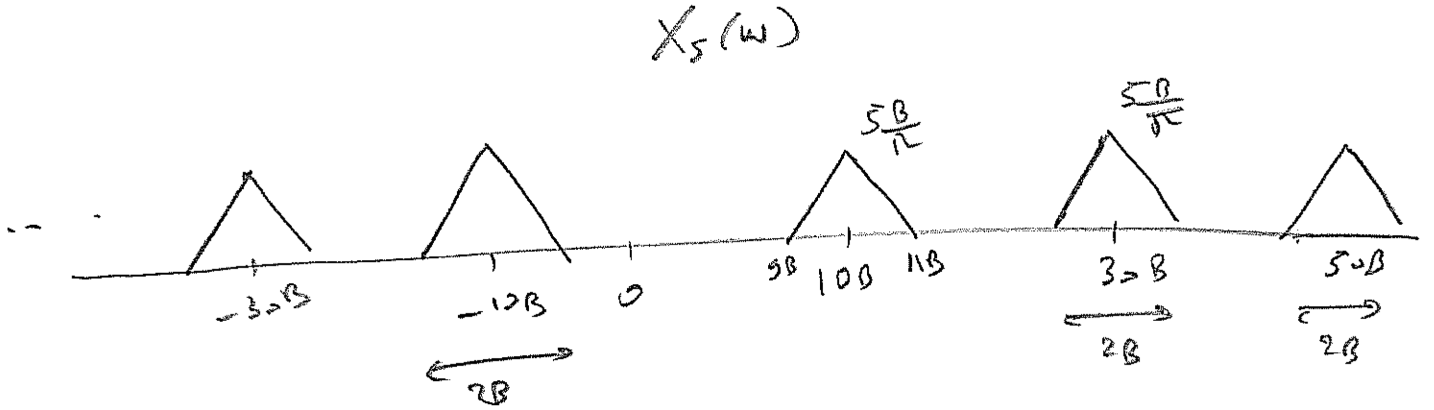
c) (7%) Determine and sketch $X_s(w)$ the CTFT of the sampled signal $x_s(t)$.

$$x_s(t) = P(t) \cdot x(t)$$

$$X_s(w) = \frac{1}{2\pi} X(w) * P(w)$$

$$= \frac{1}{2\pi} X(w) * 10B \left[\sum \delta(w - 10B - 20Bk) + \delta(w + 10B - 20Bk) \right]$$

$$= \frac{5B}{\pi} \sum_{k=-\infty}^{\infty} X(w - 10B - 20Bk) + X(w + 10B - 20Bk)$$



Assume that I would like now to use this sampler as an AM modulator at a frequency of $w_a = 25$ B rad/sec. This could be done using the system shown in figure 9 below.

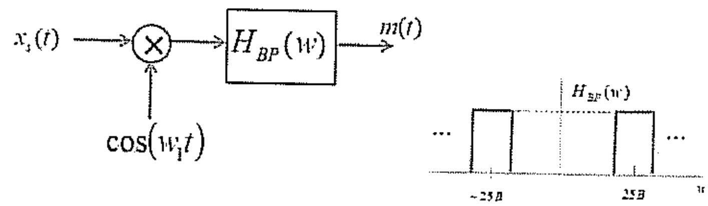
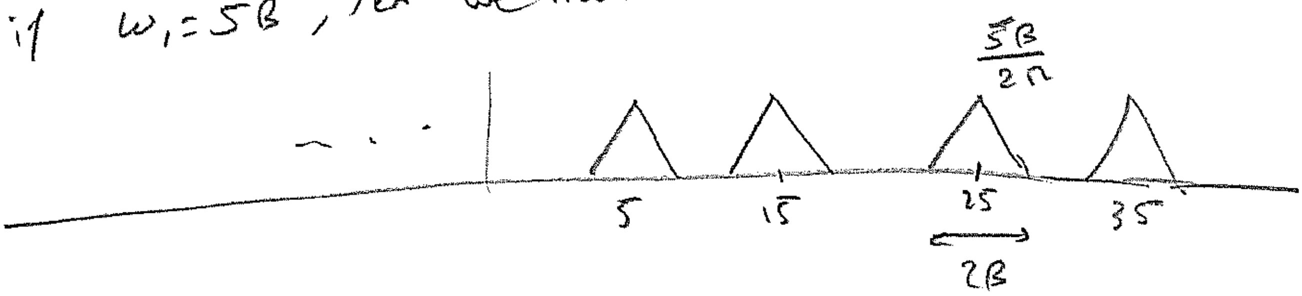


Figure 9:

d) (6%) Determine the frequency of the oscillator w_1 which allows the signal $y(t)$ to act as a DSB AM signal.

filter is at 25B while signal is at 10B, 30B, etc

if $w_1 = 5B$, then we have



passing through $H_{BP}(w)$ will recover an AM signal.